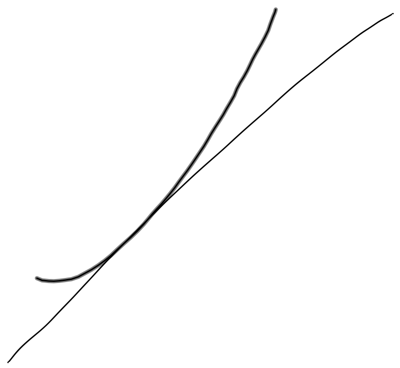


$$\cancel{\frac{x^2+2-2x^2}{(x^2+2)^2}} =$$

$$\frac{-a+b}{(a+b)^2}$$

| $a$ | $b$ | | |
|---|---|---|---|
| 100 | 5 | $\frac{-95}{(105)^2}$ | $\frac{-1}{a+b}$ $\frac{-1}{105}$ |

$$\frac{-x^2+2}{(x^2+2)^2} = -(x^2+2)$$

$$-x^2+2$$

Car tower
get out
go home
~~eat muffins~~

→ FAIL ↓

5.1/18

$$y = \frac{x}{x^2+2} \qquad \frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

$$y' = \frac{(1)(x^2+2) - x(2x)}{(x^2+2)^2}$$

sign ch

top = 0

bot = 0

$$= \frac{-x^2+2}{(x^2+2)^2} \qquad \frac{1}{x+2} - \frac{2x^2}{(x^2+2)^2}$$

$$0 \qquad 0$$

$\cancel{\ }$

$$-x^2+2=0$$

$$2 = x^2 \quad x = \pm\sqrt{2}$$

inc $[-\sqrt{2}, \sqrt{2}]$

$- \ - \ - \mid + + + + \mid - - -$

$-10 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 10$

dec $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

$x \le -\sqrt{2}$ and $x \ge +\sqrt{2}$

$$y' = \frac{-x^2+2}{(x^2+2)^2}$$

$$y'' = \frac{(-2x)(x^2+2)^2 - (-x^2+2)[2(x^2+2)(2x)]}{((x^2+2)^2)^2}$$

$$= \frac{2x(x^2+2)[-(x^2+2) - 2(2-x^2)]}{(x^2+2)^4}$$

$x=0$

$x = \pm\sqrt{6}$

$$= \frac{2x(x^2+2)[x^2-6]}{(x^2+2)^4} = 0$$

$\neq 0$

C-DN   C-UP   C-DN   C-UP

$+ - + -$

$- \ - \ - \mid + + + \mid - \ - \ - \mid + + +$

$-10 \quad -\sqrt{6} \quad -1 \quad 0 \quad 1 \quad \sqrt{6} \quad 10$

0 POI   0 POI   0 POI

C-DN: $(-\infty, -\sqrt{6}) \cup (0, \sqrt{6})$

C-UP:

5.1/16 $\quad x^4 - 8x^2 + 16$

$y' : 4x^3 - 16x = 4x(x^2-4)$

$x = 0, -2, +2 \qquad = 4x(x-2)(x+2)$

$+ - - - \qquad + - - +$

$- - - \mid + + + \mid - - \mid + + +$

$-10 \quad -2 \quad -1 \quad 0 \quad \frac{\pi}{2} \quad +2 \quad 10$

inc: $[-2, 0] \cup [2, \infty)$

dec: $(-\infty, -2] \cup [0, 2]$

$y' = 4x^3 - 16x$

$y'' = 12x^2 - 16 = 12\left(x^2 - \frac{4}{3}\right)$

$\Rightarrow x = \pm\sqrt{\frac{4}{3}}$

$+ + + \mid - - - \mid + + +$

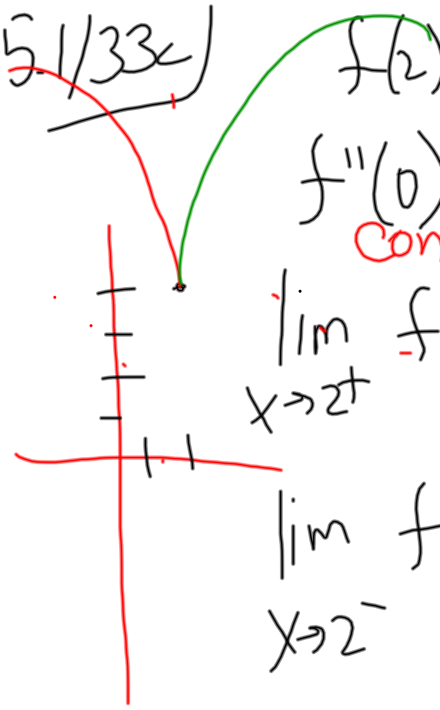
$-100 \quad -\sqrt{\frac{4}{3}} \quad 0 \quad +\sqrt{\frac{4}{3}} \quad 10$

5.1/33c)

$f(2) = 4$ $(2,4)$

$f''(0) < 0$ when $x \neq 2$ *

Concave dn everywhere*



$\lim_{x \to 2^+} f'(x) = +\infty$

$\lim_{x \to 2^-} f'(x) = -\infty$

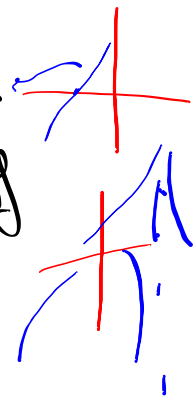
*(except 2)

## technique

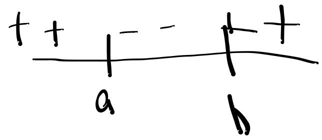
### creating a sign chart for $f(x)$

Purpose: find intervals on which $f(x)$ pos or neg

- identify "cut points"
  - x values that are zeros
  - x values that are pts of discontinuity



- "plot" cut points on number line



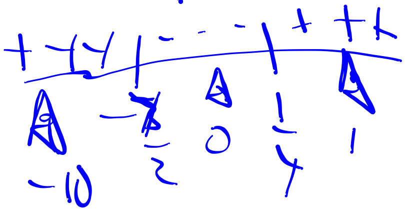
- those cut points "cut" the number line into intervals
  the functions value on a particular interval are all positive or all negative
- So, check 1 pt in each interval to see what SIGN is

ex:

$$\frac{4x-1}{2x+7} > 0$$

$4x-1=0 \quad x=\frac{1}{4}$

$2x+7=0 \quad x=-\frac{7}{2}$



identify where $f^{ns}$ are increasing
or decreasing (I or D)

I or D is a FIRST DERIVATIVE property

⇝

- Find First Deriv. $f'(x)$
- Create a sign chart of $f'(x)$
- interpret sign chart
  - * $f'(x)$ positive ⇒ $f(x)$ increasing
  - * $f'(x)$ negative ⇒ $f(x)$ decreasing

the "cut points" from the sign chart are called
CRITICAL POINTS (if they are pts)
& CRITICAL VALUES (CRITICAL NUMBERS) } if they are x-values

identify where fns $f(x)$ are concave up or concave down

2 Concavity is a $2^{ND}$ Derivative property

2 Find $2^{ND}$ derivative

Create a sign chart of $f''(x)$

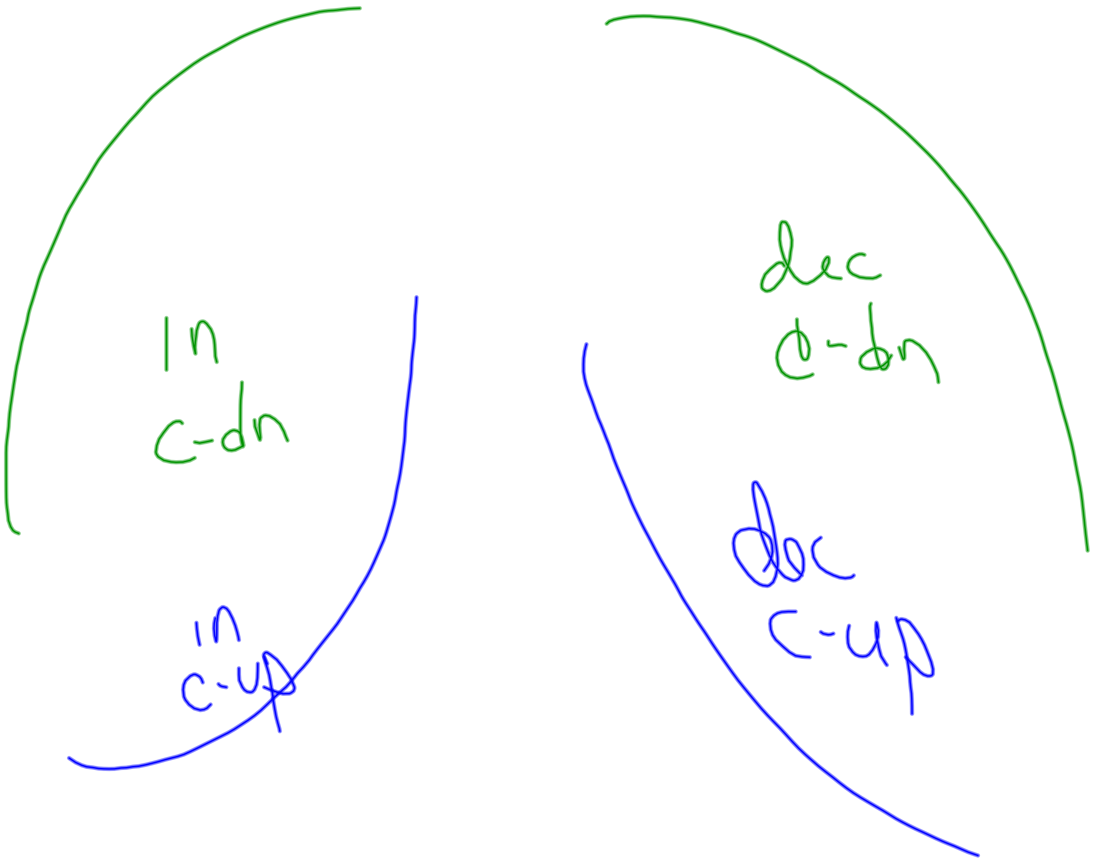
Interpret sign chart

* on intervals where $f''(x)$ is POSITIVE
  $\Rightarrow f(x)$ is concave up
  $\Rightarrow f'(x)$ is INCREASING
* on intervals where $f''(x)$ is NEGATIVE
  $\Rightarrow f(x)$ is concave down
  $\Rightarrow f'(x)$ is DECREASING

2 the "cut points" from the sign chart are called POTENTIAL INFLECTION POINTS (if they are pts)

AND ARE ONLY called POINTS OF INFLECTION

if * concavity changes
* $f^n$ is continuous


In
c-dn
in
c-up
dec
c-dn
dec
c-up

5.1/18)

$$f(x) = \frac{x}{x^2+2}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

$$f'(x) = \frac{(1)(x^2+2) - x(2x)}{(x^2+2)^2}$$

$$= \frac{-x^2+2}{(x^2+2)^2}$$

zeros

$-x^2+2=0$

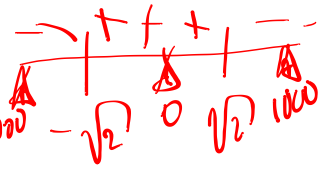
$x^2=2$

$x=\pm\sqrt{2}$

denom never zero

inc: $[-\sqrt{2},\sqrt{2}]$

dec $(-\infty,-\sqrt{2}]\cup[\sqrt{2},\infty)$




$$f'(x) = \frac{-x^2+2}{(x^2+2)^2}$$

$$f''(x) = \frac{(-2x)(x^2+2)^2 - (-x^2+2)(2(x^2+2)(2x))}{((x^2+2)^2)^2}$$

$$= \frac{2x(x^2+2)\left[-(x^2+2) - (-x^2+2)(2)\right]}{(x^2+2)^4}$$

$$= \frac{2x(x^2+2)[x^2-6]}{}$$

$$= \frac{2x(x^2+2)(x-\sqrt{6})(x+\sqrt{6})}{(x^2+2)^4}$$

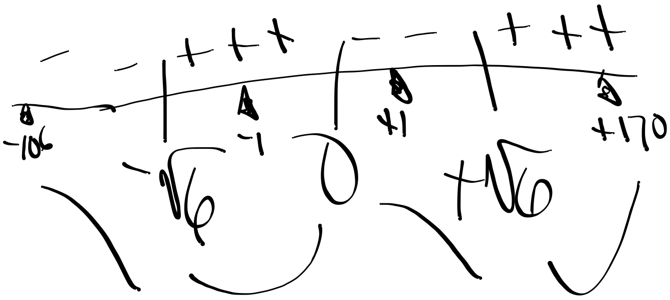
zeros

$x=0$

$x=-\sqrt{6}$

$x=+\sqrt{6}$

den nothing



5.1/24)

$$y = x e^{x^2} \qquad (1)(e^{x^2}) + (x)(2x e^{x^2})$$

$$y' = \underline{e^{x^2}} + 2x \cdot x \cdot \underline{e^{x^2}} = 0$$

$$(-\infty, \infty) \qquad e^{x^2}(1+2x^2) > 0$$

$$y' = e^{x^2}(1+2x^2)$$

$$y'' = (2x e^{x^2})(1+2x^2) + (e^{x^2})(4x)$$

$$= 2x e^{x^2}[1+2x^2+2]$$

$$= 2x e^{x^2}[2x^2+3) = 0$$

$x=0$

$x=0 \quad \cancel{\emptyset} \quad \cancel{\emptyset}$

$- - - - \; | \; + + + +$

$0$

c-up: $(0,\infty)$

c-dn: $(-\infty,0)$

POI: @ $x=0$

Vote.eschew.org/
Middletown/
Superlatives

5.1/26 $y = x^2 \ln x$

$$y' = 2x \ln x + \frac{x^2}{x} =$$

$$2x \ln x + x = x(2\ln x + 1)$$

$y' = 0$ when $x = 0$

$2\ln x + 1 = 0$

$2\ln x = -1$

OR

$\ln x = -\frac{1}{2}$

i.e. $x = e^{-1/2} = \frac{1}{\sqrt{e}}$